Miami: Geometry and Physics 2016

Abelian and Discrete Symmetries in F-theory Compactifications

Mirjam Cvetič

Topics, closely related to Ron’s numerous contributions at the interface with physics, including recent ``mini-revolution” in F-theory compactification.

As a colleague at Penn, he is generous and patient, teaching physicists, including myself, aspects of algebraic geometry.

Resulted in my collaboration with him on heterotic string theory & F-theory, including one topic of this talk.

Wishing you a Happy 60th Birthday!

Outline:

I. Motivation: F-theory & Particle Physics

Geometry of Singular Elliptically Fibered Calabi-Yau Manifolds

building blocks → non-Abelian gauge symmetry, matter, coupl.

particle physics models→ highlight 3-family Standard Model

II. Formal Developments:

Abelian Symmetries $U(1)^n$ – elliptic fibrations with rational sections & Mordell Weil group

Discrete Symmetries Z_n – genus-one fibrations with multi-sections & Tate-Shafarevich group

Overview – Upenn-centric

F-Theory Compactifications with Abelian Symmetries

[Elliptic Fibrations with n-rational sections - $U(1)^n$ ←→ rnk n Mordell-Weil (MW) Group]

Based on:

arXiv:1303.6970 [hep-th]: M. C., Denis Klevers, Hernan Piragua
arXiv:1307.6425 [hep-th]: M. C., D. Klevers, H. Piragua
(rnk 2 MW on Calabi-Yau threefolds → $U(1)^2$)

arXiv:1306.0236 [hep-th]: M. C., Antonella Grassi, D. Klevers, H. Piragua
(rnk 2 MW on Calabi-Yau fourfolds; inclusion of flux)

arXiv:1310.0463 [hep-th]: M.C., D. Klevers, H. Piragua, Peng Song
(rnk 3 MW)

arXiv:1503.02068 [hep-th]: M.C., D.Klevers, Damian. M.Peña, P. Konstantin Oehlmann , Jonas Reuter
(particle physics models)

arXiv:1507.05954 [hep-th]: M.C. D. Klevers, H. Piragua, Wati Taylor
(rnk 2 MW and ``un-Higgsing'')

⇕ related

F-Theory Compactifications with Discrete Symmetries

[Elliptic Fibrations with n-section Z_n ←→ rnk n Tate-Shafarevich (TS) Group]

Based on:

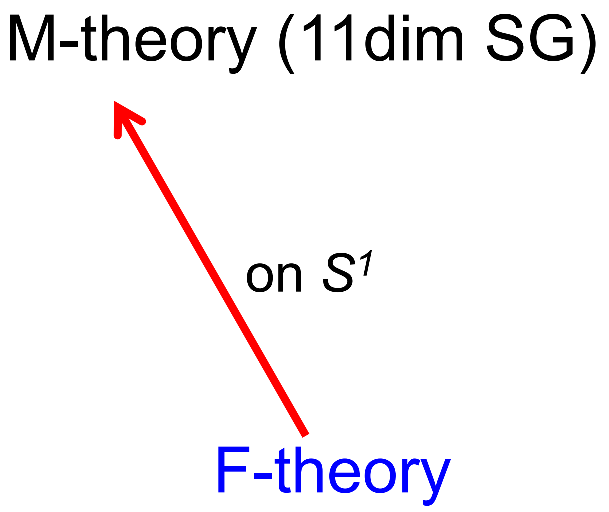
arXiv:1502.6970[hep-th]:
M. C., Ron Donagi, Denis Klevers, Hernan Piragua, Maximilian Poretschkin
(rnk 3 TS on Calabi-Yau threefolds → Z_3)

F-theory?

F-theory	=	Type IIB String
• coupling g_s part of geometry (12dim)		• back-reacted (p,q) 7-branes
• elliptically fibered Calabi-Yau manifold		• regions with large g_s on non-CY space

g_s –string coupling

F-theory?



M-theory (11dim SG)

on S^1

F-theory = Type IIB

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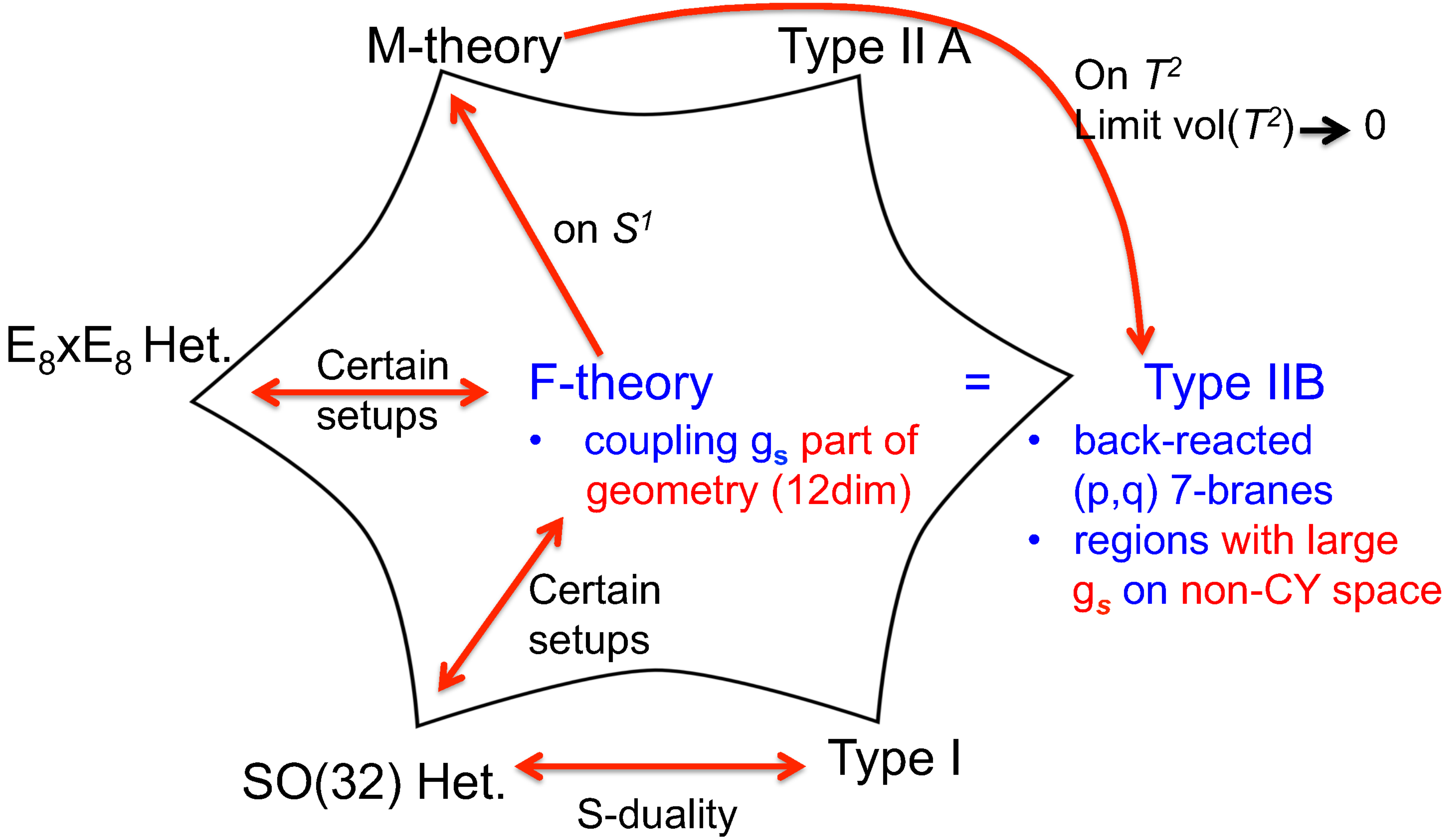


M-theory

Type II A

On T^2

Limit vol(T^2) → 0

on S^1

E_8xE_8 Het.

Certain setups

F-theory = Type IIB

- coupling g_s part of geometry (12dim)

- back-reacted (p,q) 7-branes
- regions with large g_s on non-CY space

Certain setups

SO(32) Het.

S-duality

Type I

F-theory & Particle Physics

MOTIVATION

F-Theory Motivation

Physics: a broad domain of non-perturbative string theory landscape with new promising particle physics & cosmology

- SU(5) GUT couplings that are absent in perturbative string theory w/ D-branes, e.g.,10 10 5
- appearance of exceptional gauge symmetries (E_6)

[Donagi,Wijnholt'08]
[Beasley,Heckman,Vafa'08]....

Geometry: geometric description at large string coupling in terms singular elliptically fibered Calabi-Yau manifolds

- Determine discrete data:
  gauge symmetry, matter reps. & multiplicities, Yukawa couplings

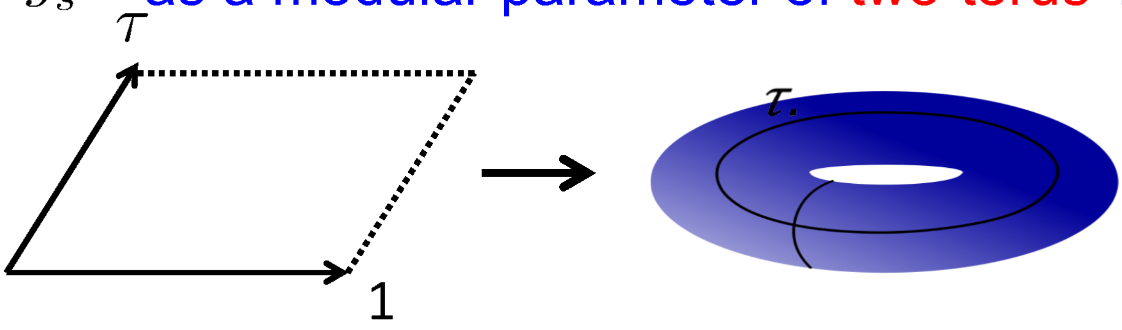
Type IIB perspective

F-THEORY BASIC INGREDIENTS

F-theory Compactification: Basic Ingredients

F-theory geometrizes the (Type IIB) string coupling (axio-dilaton)

$\tau \equiv C_0 + i g_s^{-1}$ as a modular parameter of two-torus $T^2(\tau)$ w/SL(2,Z)



[Vafa]

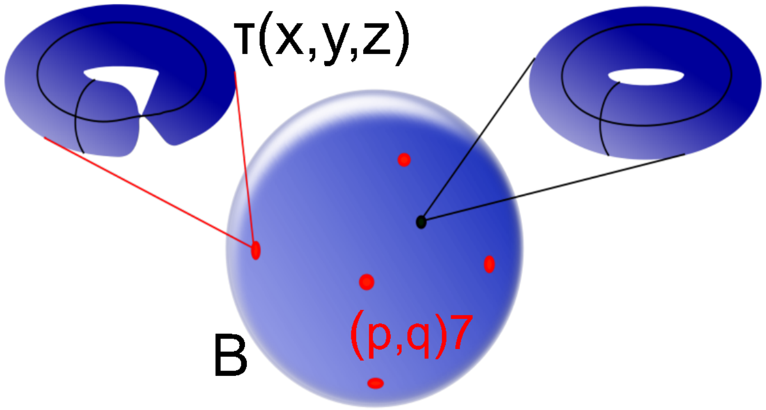
Compactifcation is a two-torus $T^2(\tau)$-fibration over a compact base space B:

Weierstrass form:

$$y^2 = x^3 + f x z^4 + g z^6$$

f, g- function fields on B

[z:x:y] homog. coords on **P²**(1,2,3)



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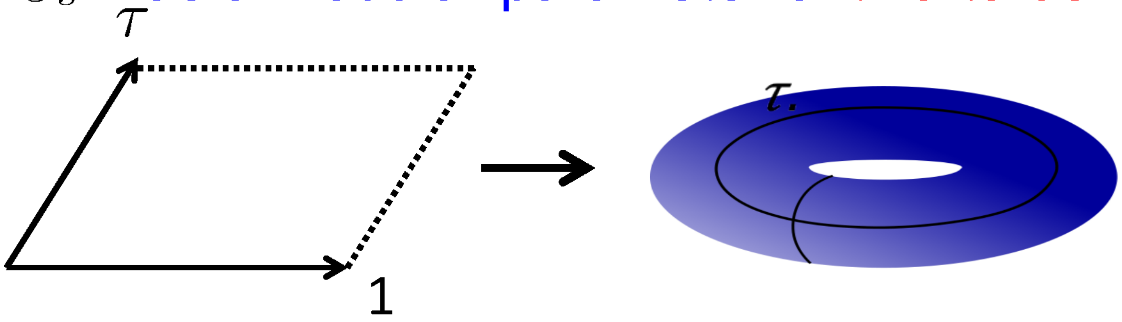

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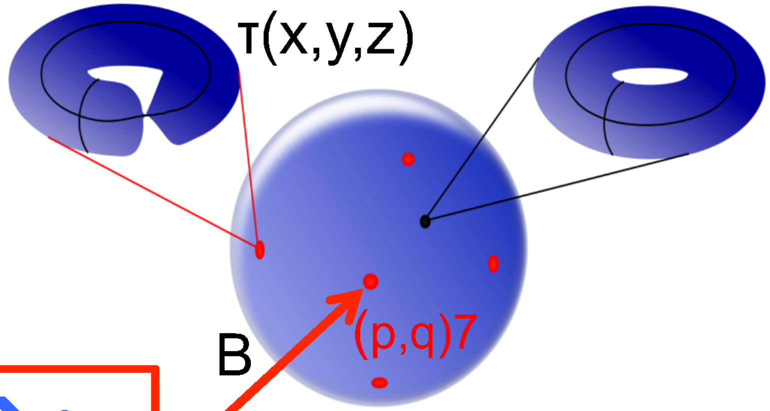


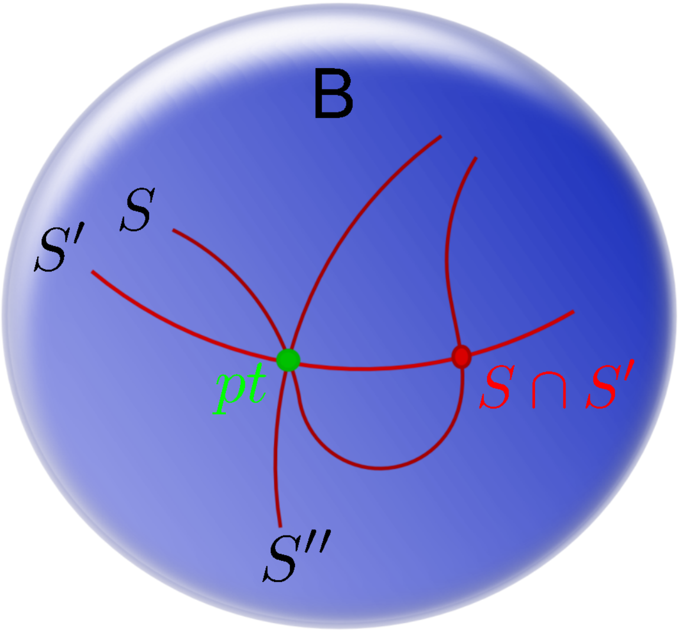
singular $T^2(\tau)$-fibr. → $g_s \rightarrow \infty$
location of (p,q) 7-branes

F-theory: basic ingredients

- Total space of torus-fibration: singular elliptic Calabi-Yau manifold X

  D=4, N=1 vacua: fourfold X_4 [all dimensions complex]

- Singularities encode complicated set-up of intersecting D-branes:



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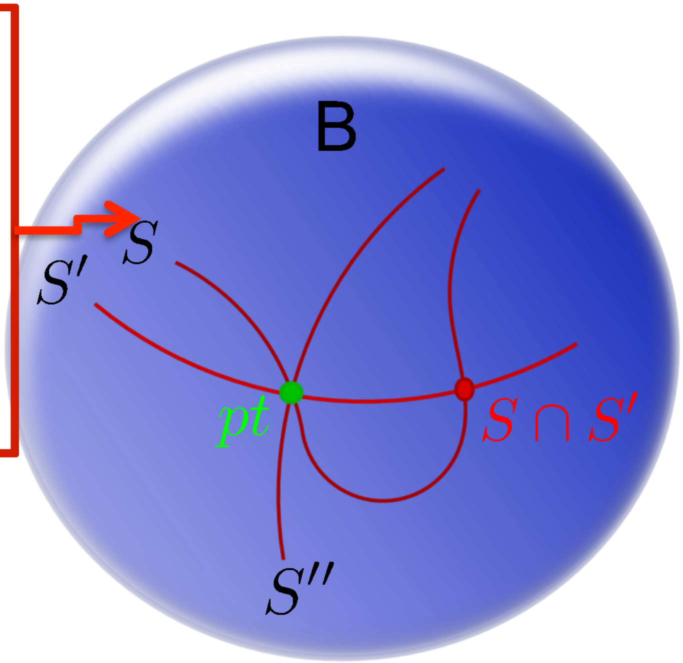
Non-Abelian gauge symmetry in codimension-one (ADE) singul. in B (divisors wrapped by branes)



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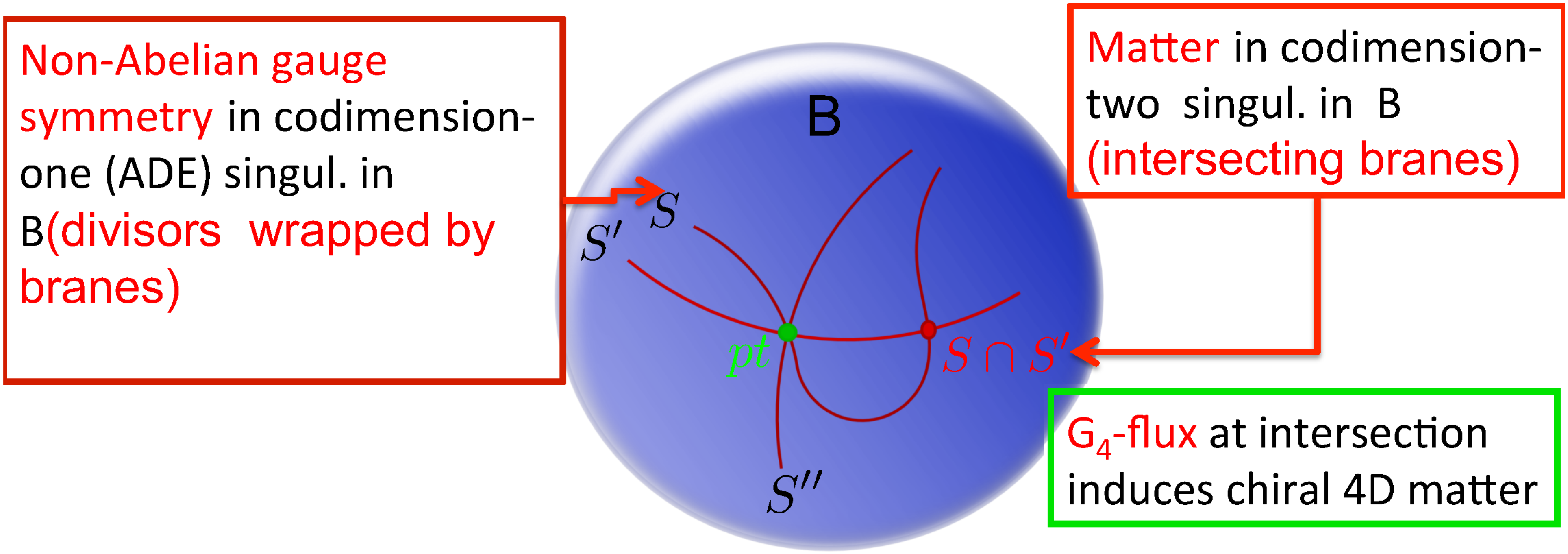

Non-Abelian gauge symmetry in codimension-one (ADE) singul. in B(divisors wrapped by branes)

Matter in codimension-two singul. in B (intersecting branes)

G_4-flux at intersection induces chiral 4D matter

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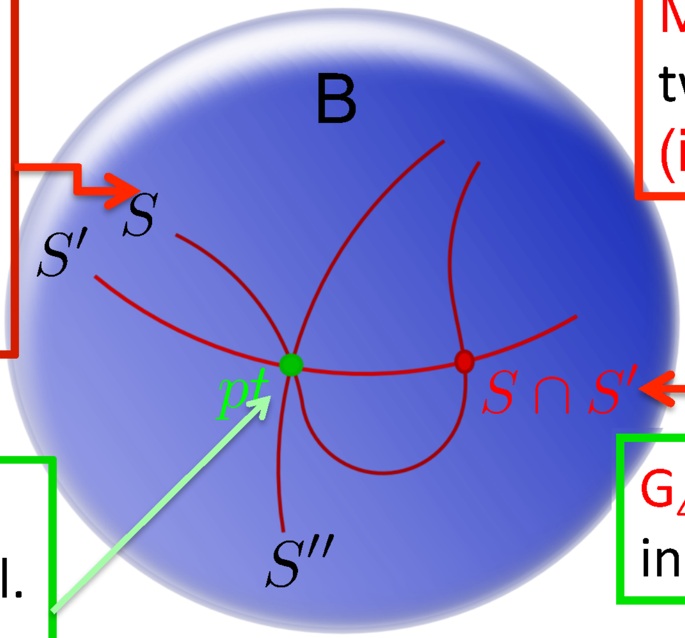


Non-Abelian gauge symmetry in codimension-one (ADE) singul. in B (divisors wrapped by branes)

Matter in codimension-two singul. in B (intersecting branes)

Yukawa couplings in codimension-three singul. in B $pt = S \cap S' \cap S''$

G_4-flux at intersection induces chiral 4D matter

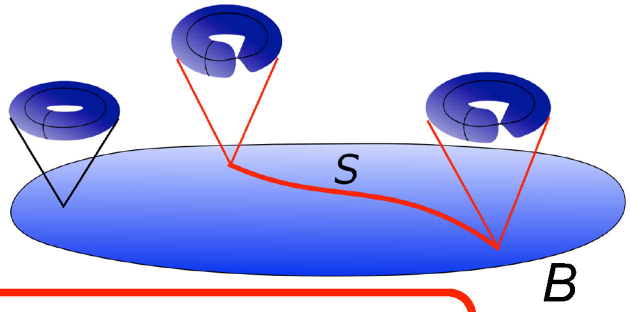
Highlights: Non-Abelian Gauge Symmetry

[Kodaira; Tate; Vafa; Morrison,Vafa;…]

1. Weierstrass form for elliptic fibration of *X*

$$y^2 = x^3 + fxz^4 + gz^6$$

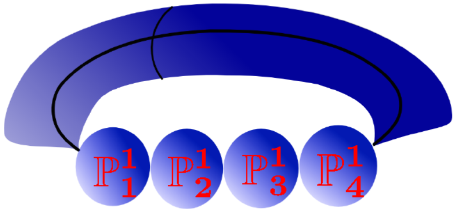
2. Severity of singularity along divisor S in B:



[$ord_S(f), ord_S(g), ord_S(\Delta)$] ←→ Singularity type of fibration of *X*

3. Resolution: singularity type ←→ structure of a tree of $\mathbb{P}^1$ ‘s over *S*

I_n-singularity ←→ SU(n) Dynkin diagram



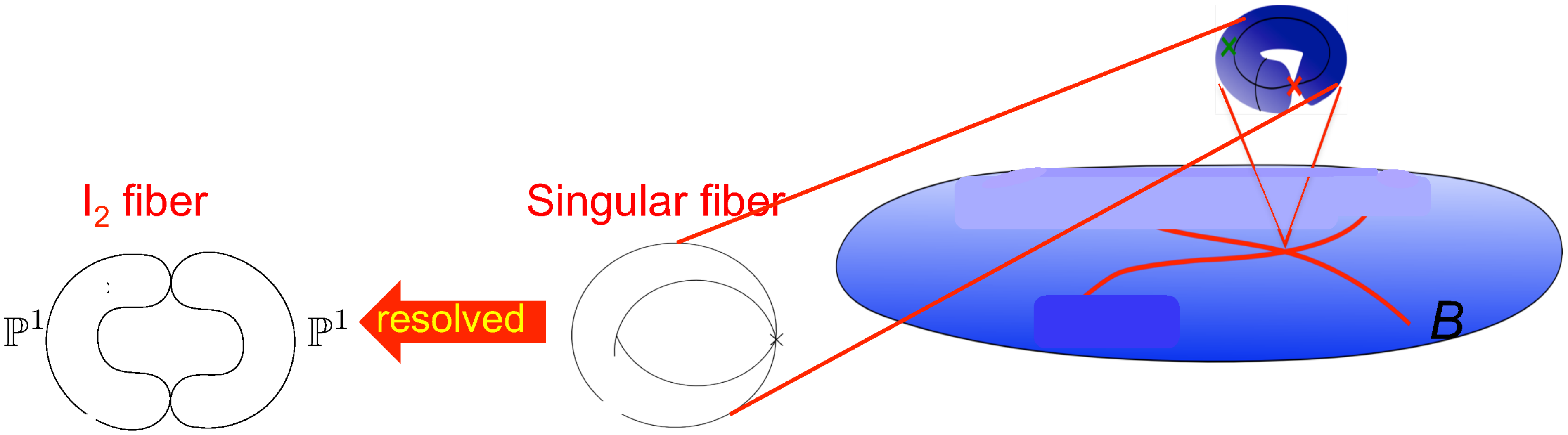
- Cartan generators for A^i gauge bosons: in M-theory via Kaluza-Klein (KK) reduction of C_3 potential along (1,1)-forms $\omega_i \leftrightarrow \mathbb{P}^1_i$ on resolved X

$$C_3 \supset A^i \omega_i$$

- Non-Abelian generators: light M2-brane excitations on $\mathbb{P}^1$‘s [Witten]

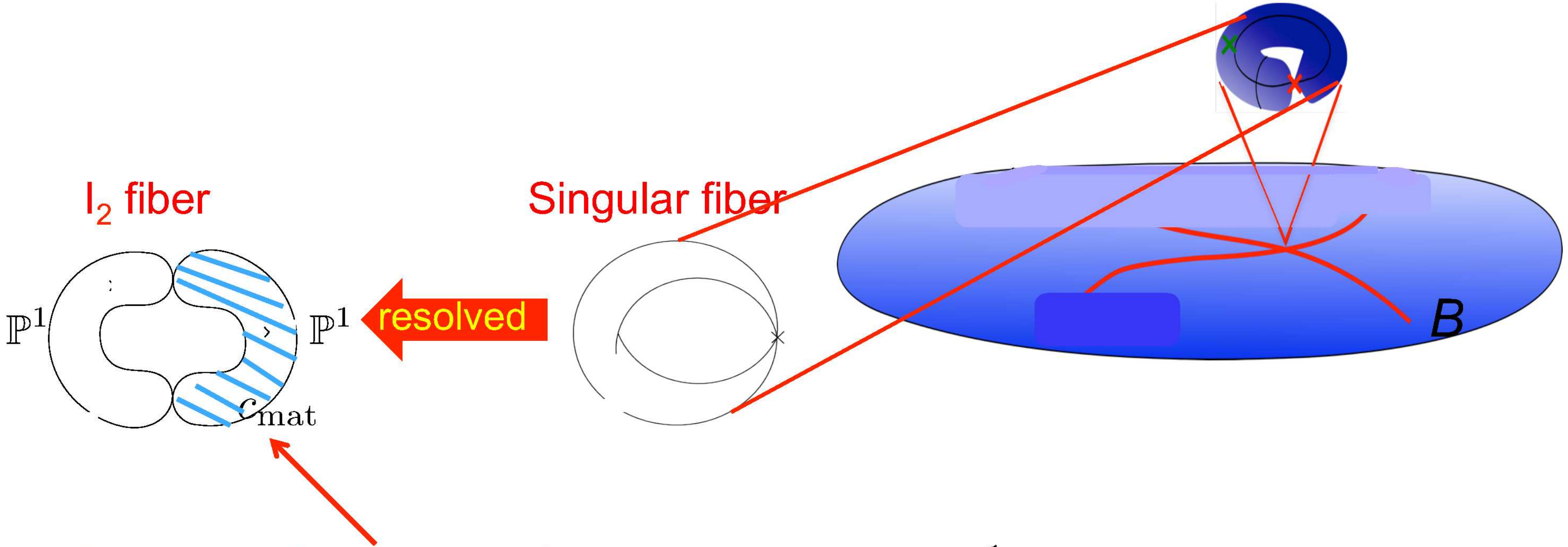
Highlights: Matter

Singularity at codimension-two in *B*:



Highlights: Matter

Singularity at codimension-two in *B*:



w/isolated (M2-matter) curve wrapping $\mathbb{P}^1$ → charged matter
(determined via intersection theory)

Initial focus: F-theory with SU(5) Grand Unification

[Donagi,Wijnholt'08][Beasley,Heckman,Vafa'08]...

Model Constructions:

local [Donagi,Wijnholt'09-10]...[Marsano,Schäfer-Nameki,Saulina'09-11]...

Review: [Heckman]

global

[Blumehagen,Grimm,Jurke,Weigand'09][M.C., Garcia-Etxebarria,Halverson'10]...
[Marsano,Schäfer-Nameki'11-12]...[Clemens,Marsano,Pantev,Raby,Tseng '12]...

c.f., Clemens's talk

Recent progress on other Particle Physics Models:

Standard Model building blocks (via toric techniques) [Lin,Weigand'14]

First Global 3-family Standard, Pati-Salam, Trinification Models
[M.C., Klevers, Peña, Oehlmann, Reuter 1503.02068]

➡ highlights

I. Particle Physics & F-theory

concrete examples

Torus Fibrations via Toric Geometry Techniques:

i. Torus = elliptic curve $\mathcal{C}_{F_i}$ as a Calabi-Yau hypersurface in the two-dimensional toric variety $\mathbb{P}_{F_i}$ (associated with 16 reflexive polytops F_i):

$$\mathcal{C}_{F_i} = \{p_{F_i} = 0\} \text{ in } \mathbb{P}_{F_i}$$

[Toric variety $\mathbb{P}_{F_i}$ as a generalized projective space $\mathbb{P}_{F_i} = \frac{\mathbb{C}^{m+2}\backslash \mathrm{SR}}{(\mathbb{C}^*)^m}$

(blow-ups of $\mathbb{P}^2$) w/ $\mathcal{C}_{F_i}$ as anti-canonical divisor in $\mathbb{P}_{F_i}$]

[Klevers, Peña, Piragua, Oehlmann, Reuter'14]

ii. Elliptically fibered Calabi-Yau space X_{F_i}

Impose Calabi –Yau condition
w/ coordinates in $\mathbb{P}_{F_i}$ and coeffs. of $\mathcal{C}_{F_i}$ lifted to sections
of line bundles of certain degree in B

$$\begin{tikzcd} \mathcal{C}_{F_i} \subset \mathbb{P}_{F_i} \arrow[r] & X_{F_i} \arrow[d] \\ & B \end{tikzcd}$$

Fibration determined by two divisors $\mathcal{S}_7, \mathcal{S}_9,$ on B

Model Building Strategy:

i. Construction of X_{F_i} w/ Particle Physics

gauge symmetry (codim-1), matter reps. (codim-2) & Yukawas (codim-3)

F_{11} - Standard Model $\mathbf{SU(3)} \times \mathbf{SU(2)} \times \mathbf{U(1)}$ focus

Representation	$(\mathbf{3},\mathbf{2})_{1/6}$	$(\bar{\mathbf{3}},\mathbf{1})_{-2/3}$	$(\bar{\mathbf{3}},\mathbf{1})_{1/3}$	$(\mathbf{1},\mathbf{2})_{-1/2}$	$(\mathbf{1},\mathbf{1})_{-1}$

$$p_{F_{11}} = s_1 e_1^2 e_2^2 e_3 e_4^4 u^3 + s_2 e_1 e_2^2 e_3^2 e_4^2 u^2 v + s_3 e_2^2 e_3^2 u v^2 + s_5 e_1^2 e_2 e_4^3 u^2 w + s_6 e_1 e_2 e_3 e_4 u v w + s_9 e_1 v w^2$$

[hypersurface constraint in dP_4 ($\mathbb{P}^2[u:v:w]$ with four blow-ups $[e_1:e_2:e_3:e_4]$])

F_{13} - Pati-Salam Model

F_{16} - Trinification Model

ii. Chiral index for D=4 matter:

$$\begin{tikzcd} c_{mat} \arrow[r] & \boxed{\mathcal{C}_{\mathbf{R}}} \arrow[d] \\ & \Sigma_{\mathbf{R}} \end{tikzcd}$$

$$\chi(\mathbf{R}) = \int_{\mathcal{C}_{\mathbf{R}}} G_4$$

a) construct G_4 flux by computing $H_V^{(2,2)}(\hat{X})$

b) determine matter surface $\mathcal{C}_{\mathbf{R}}$ (via resultant techniques)

iii. Global consistency – D3 tadpole cancellation (Gauss law):

$$\frac{\chi(X)}{24} = n_{\mathrm{D3}} + \frac{1}{2}\int_X G_4 \wedge G_4$$

a) satisfied for integer and positive n_{D3}

b) check, all quantum field theory anomalies are cancelled

Standard Model:

Base B = $\mathbb{P}^3$ Divisors in the base:

$$\mathcal{S}_7 = n_7 H_{\mathbb{P}^3}$$
$$\mathcal{S}_9 = n_9 H_{\mathbb{P}^3}$$

Solutions (#(families);n_{D3}) for allowed (n_7,n_9):

$n_7 \backslash n_9$	1	2	3	4	5	6	7
7	–	$(27;16)$	–	–			
6	–	$(12;81)$	$(21;42)$	–	–		
5	–	–	$(12;57)$	$(30;8)$	–	$(3;46)$	
4	$(42;4)$	–	$(30;32)$	–	–	–	–
3	–	$(21;72)$	–	–	–	$(15;30)$	
2	$(45;16)$	$(24;79)$	$(21;66)$	$(24;44)$	$(3;64)$		
1	–	–	–	–			
0	–	–	$(12;112)$				
-1	$(36;91)$	$(33;74)$					
-2	–						

II. U(1)-Symmetries in F-Theory

Abelian Symmetries in F-theory

Physics: important ingredient of the Standard Model and beyond

➡ Multiple U(1)'s desirable

Geometry: new CY elliptic fibrations with rational sections

While non-Abelian symmetries extensively studied ('96…)
[Kodaira; Tate;Morrison,Vafa; Bershadsky,Intriligator,Kachru,Morrison,Sadov,Vafa; Candelas,Font,...]

:

Abelian sector rather unexplored

A lot of recent progress '12-'15: [Grimm,Weigand;... Morrison,Park; M.C.,Grimm,Klevers;... Borchmann,Mayrhofer,Palti,Weigand; M.C.,Klevers,Piragua; MC,Grassi,Klevers,Piragua;... Braun,Grimm,Keitel; ... M.C.,Klevers,Piragua,Song;...Morrison,Taylor;... M.C.,Klevers,Piragua,Taylor]

U(1)’s-Abelian Symmetry

- U(1)’s gauge bosons A^m should also arise via KK reduction $C_3 \supset A^m\omega_m$.

(1,1)-forms on X

- Forbid non-Abelian enhancement (by M2’s wrapping $\mathbb{P}^1$‘s): only I_1-fibers

U(1)'s-Abelian Symmetry & Rational Sections

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(1,1)-forms on X

- Forbid non-Abelian enhancement (by M2's wrapping $\mathbb{P}^1$'s): only I_1-fibers

(1,1) - form ω_m ⟺ rational section

[Morrison,Vafa]

U(1)'s-Abelian Symmetry &Mordell-Weil Group

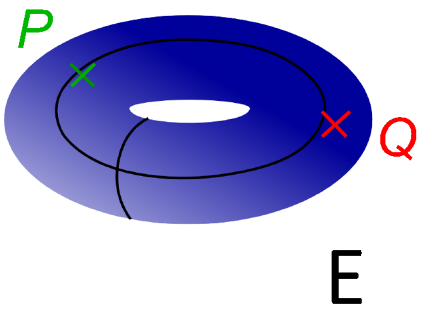
rational sections of elliptic fibr. ➡ rational points of elliptic curve

1. Rational point Q on elliptic curve E with zero point P
   - is solution (x_Q, y_Q, z_Q) in field K of Weierstrass form

$$y^2 = x^3 + fxz^4 + gz^6$$

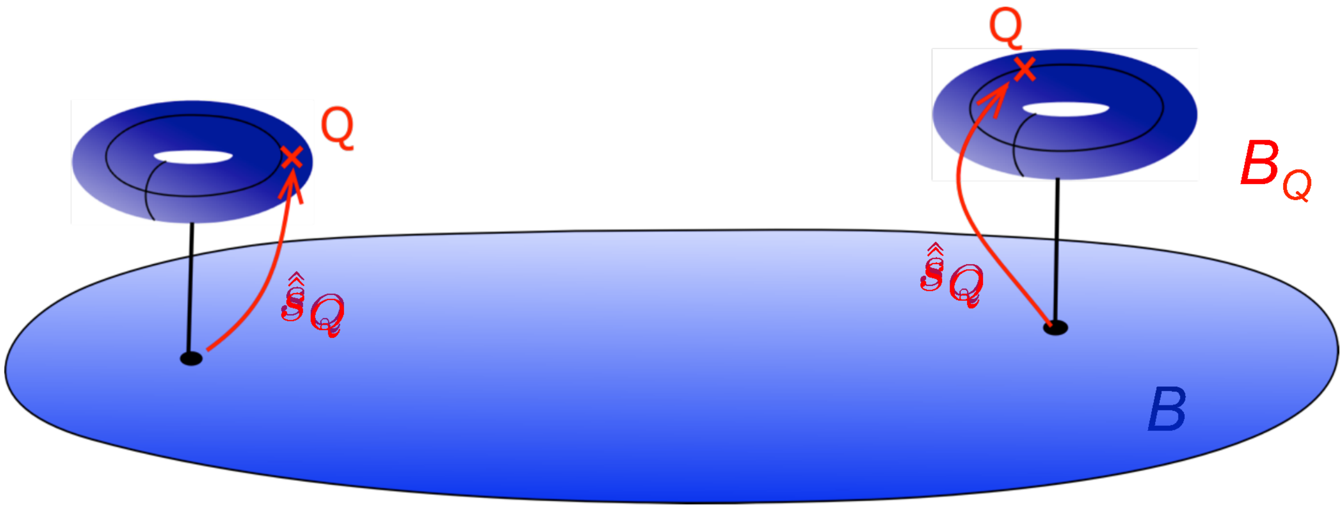
   - Rational points form group (addition) on E

➡ Mordell-Weil group of rational points



U(1)'s-Abelian Symmetry &Mordell-Weil Group

Point Q induces a rational section $\hat{s}_Q : B \to X$ of elliptic fibration



➡ $\hat{s}_Q$ gives rise to a second copy of *B* in *X*:

new divisor *B*$_Q$ in *X*

U(1)'s-Abelian Symmetry &Mordell-Weil Group

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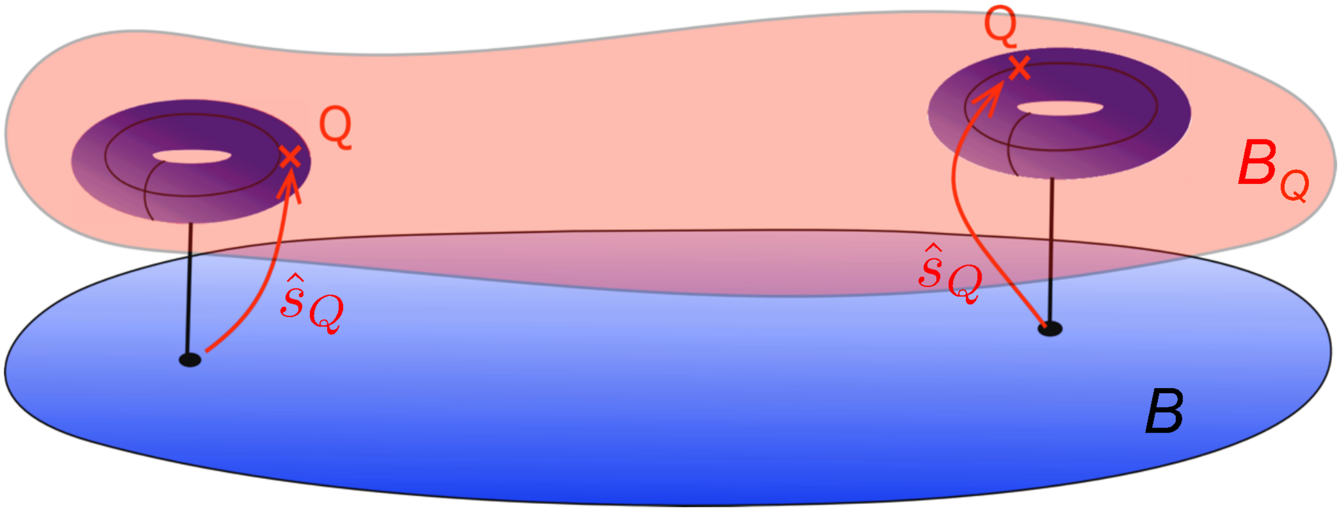


➡ $\hat{s}_Q$ gives rise to a second copy of *B* in *X*:

new divisor B_Q in *X*

➡ (1,1)-form ω_m constructed from divisor B_Q (Shioda map)

indeed (1,1) - form ω_m ⬌ rational section

Elliptic curves with rank-n Mordell-Weil group

ELLIPTIC FIBRATIONS WITH n RATIONAL SECTIONS

Elliptic Curves E with Rational Points

Elliptic curve with zero point P and n rational points Q_i

c.f., Morrison's talk

1. Consider line bundle $M=O(P+Q_1+\dots+Q_n)$ of degree $n+1$ on E:

   1) $H^0(M)=\langle x_1,\dots,x_{n+1}\rangle$, $n+1$ sections

   2) $H^0(M^k)= k(n+1)$ sections, $r := \begin{pmatrix} n+k \\ k \end{pmatrix}$ sections known (deg. k monomials in x_i)

   - $r < k(n+1)$: need to introduce new sections in M^k ($n=0, 1$)
   - $r > k(n+1)$: $r-k(n+1)$ relations between sections: E embeddable in $W\mathbb{P}^m$

2. Existence of rational points $Q_1,\dots, Q_n$:

   E non-generic Calabi-Yau one-fold in $W\mathbb{P}^m$;

   special constructions: generic Calabi-Yau in blow-up of $W\mathbb{P}^m$

   at rational points Q_i

Explicit Examples with n-rational sections – U(1)^n

elliptic curve

n=0: with P - generic CY in $\mathbb{P}^2(1,2,3)$ (Tate form)

n=1: with *P, Q* - generic CY in $\mathrm{Bl}_1\mathbb{P}^2(1,1,2)$ [Morrison,Park'12]

n=2: with *P, Q, R* - specific example: generic CY in *dP*$_2$

[Borchmann,Mayerhofer,Palti,Weigand'13;
M.C.,Klevers,Piragua 1303.6970,1307.6425;
M.C.,Grassi,Klevers,Piragua 1306.0236]

- generalization: non-generic cubic in $\mathbb{P}^2[u:v:w]$

[M.C.,Klevers,Piragua,Taylor 1507.05954]

n=3: with *P, Q, R, S* - CICY in $\mathrm{Bl}_3\mathbb{P}^3$ [M.C.,Klevers,Piragua,Song 1310.0463]

n=4 determinantal variety in $\mathbb{P}^4$ higher *n,* not clear…

…

$U(1)^2$: Concrete Example

[M.C., Klevers,Piragua]
[Borchmann,Mayrhofer,Palti,Weigand]

representation as hypersurface in dP_2 :

$$p=u(s_1u^2e_1^2e_2^2+s_2uve_1e_2^2+s_3v^2e_2^2+s_5uwe_1^2e_2+s_6vwe_1e_2+s_8w^2e_1^2)+s_7v^2we_2+s_9vw^2e_1$$

$[u{:}v{:}w{:}e_1{:}e_2]$ –homogeneous coordinates of dP_2

u v w e_1 e_2

$$\boxed{\begin{aligned} P:&\quad E_2\cap p=[-s_9:s_8:1:1:0]\,,\\ Q:&\quad E_1\cap p=[-s_7:1:s_3:0:1]\,,\\ R:&\quad D_u\cap p=[0:1:1:-s_7:s_9]\,.\end{aligned}}$$

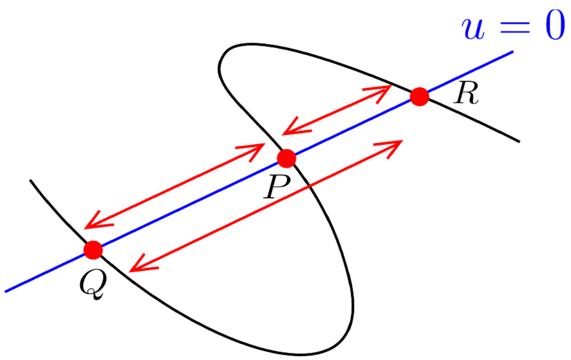
Sections represented by intersections of different divisors in dP_2 with p

generic CY in dP_2

Also determine charged matter (at co-dim. 2) and multiplicities

U(1)^2: Further Developments

General U(1)^2 construction: [M.C., Klevers, Piragua, Taylor 1507.05954]



non-generic cubic curve in $\mathbb{P}^2[u:v:w]$:

$$uf_2(u,v,w)+\prod_{i=1}^{3}(a_iv+b_iw)=0$$

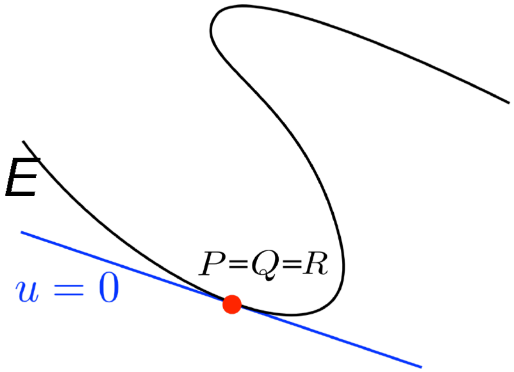
$f_2(u,v,w)$ degree two polynomial in $\mathbb{P}^2[u:v:w]$

Study of non-Abelian enhancement (unHiggsing) by merging rational points P,Q,R →

non-local horizontal divisors (Abelian) turn into local vertical ones (non-Abelian) →
both in geometry(w/ global resolutions) & field theory (Higgsing matter)

Non-Abelian Gauge Enhancement:U(1)^2

Reduce MW-rank to zero by merging rational points *Q*, *R* with zero *P*



$$uf_2(u,v,w) + \lambda_1\lambda_2(a_1v + b_1w)^3 = 0$$

- rk(MW)=2 → 1 as $\overline{PQ}$ → 0
- rk(MW)=1 → 0 as $\overline{PR}$ → 0

Tuned fibration with co-dim. 1 singularity built in:

1. U(1)*x*U(1) ➡ SU(3): set $\lambda_i = 1$ at locus $f_2(0,-b_1,a_1) = 0$ in *B*

   I_3-singularity at P

2. U(1)*x*U(1) ➡ SU(2)*x*SU(2): set $f_2(0,-b_1,a_1) = 1$

   I_2-fiber at $\lambda_i = 0$ in *B*: $uf_2(u,v,w) = 0$

3. General case not rank preserving: U(1)^2 →SU(3)*x*SU(2)^3 or a subgroup
   Byproduct: first construction of matter (at co-dim. 2 sing.) in symmetric two-index rep. of SU(3).

III. Discrete Symmetries in F-Theory

Why Discrete Symmetries in F-theory?

Physics: important ingredient of beyond the Standard Model physics

➡ forbid terms for fast proton decay and other R-parity violating terms, e.g., R-parity (Z_2), baryon triality (Z_3) and proton hexality (Z_6); family textures

Geometry: new Calabi-Yau geometries with genus-one fibrations

These geometries do not admit a section, but a multi-section

Earlier work:[Witten; deBoer, Dijkgraaf, Hori, Keurentjes, Morgan, Morrison, Sethi;...]

Recent extensive efforts'14-'15: [Braun, Morrison; Morrison, Taylor; Klevers, Mayorga-Pena, Oehlmann, Piragua, Reuter; Anderson,Garcia-Etxebarria, Grimm; Braun, Grimm, Keitel; Mayrhofer, Palti, Till, Weigand; M.C.,Donagi,Klevers,Piragua,Poretschkin; Grimm, Pugh, Regalado]

Abelian & Discrete Gauge Symmetry in F-theory

F-theory compactification with n sections (Abelian $U(1)^{(n-1)}$)

[Morrison, Taylor;
Anderson, García-Etxebarria, Grimm, Keitel;
Braun, Grimm, Keitel]

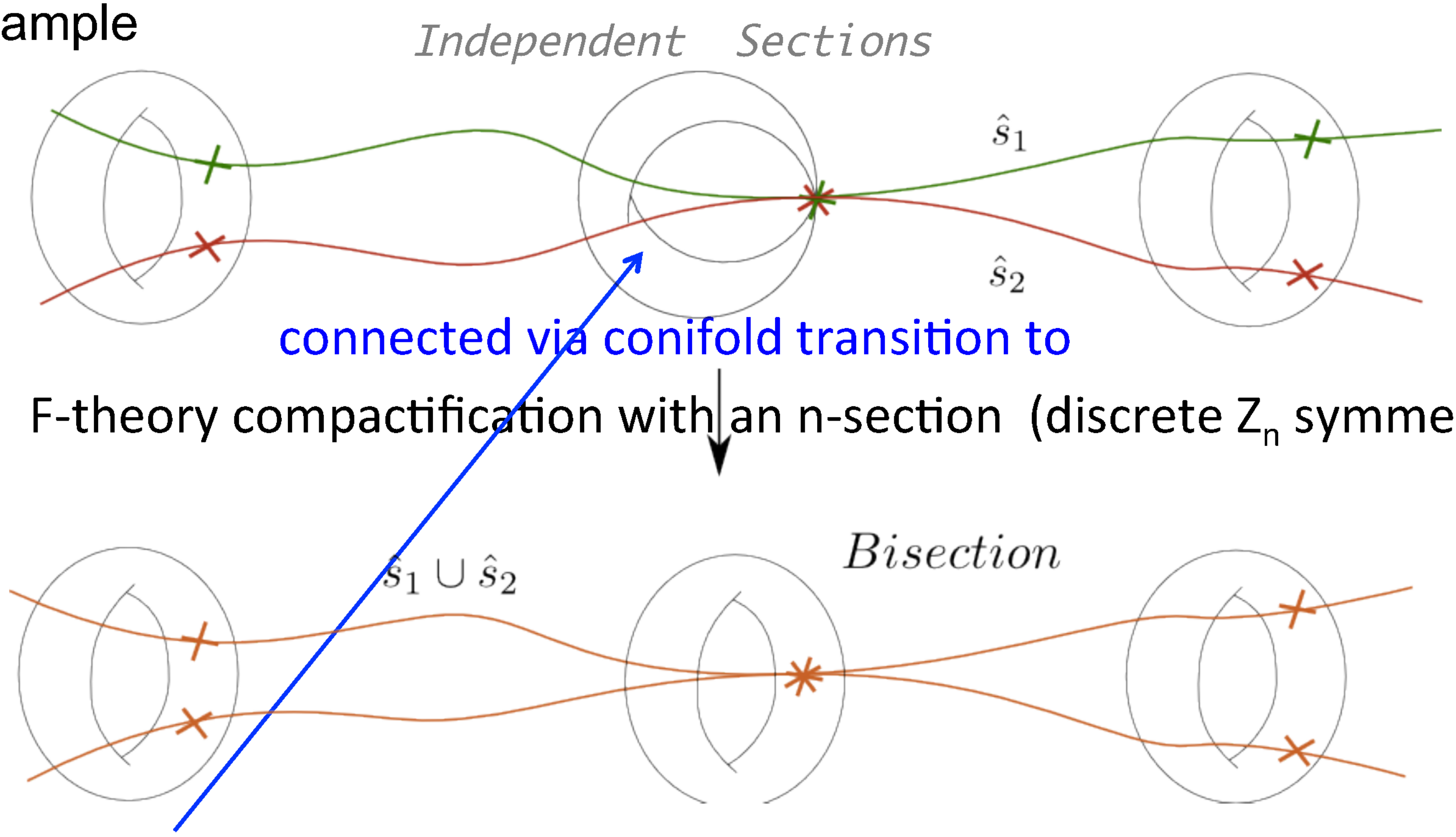
connected via conifold transition

F-theory compactification with an n-section (discrete Z_n symmetry)

Abelian & Discrete Gauge Symmetry in F-theory

F-theory compactification with n sections (Abelian $U(1)^{(n-1)}$)

n=2 example



connected via conifold transition to

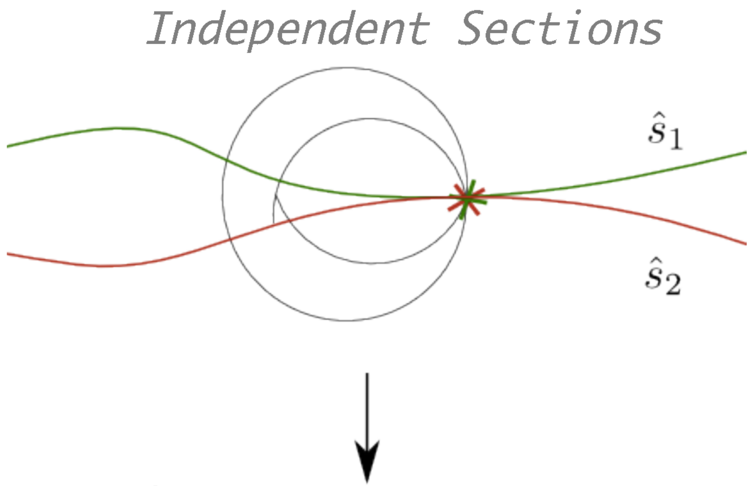
F-theory compactification with an n-section (discrete Z_n symmetry)

Torus fibration degenerates at
co-dimension 2 loci →matter

Abelian & Discrete Gauge Symmetry in F-theory

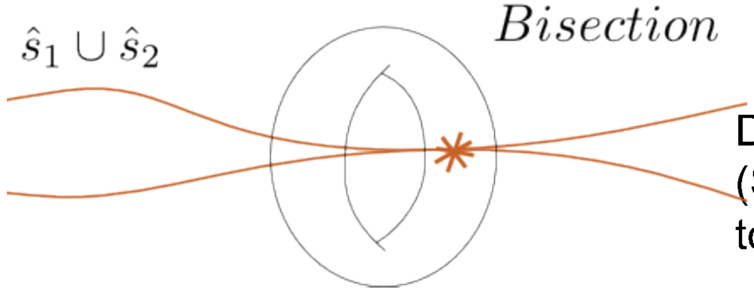
F-theory compactification with n sections (Abelian $U(1)^{(n-1)}$)

n=2 example



blow-down
(P^1 in the geometry with
multiple sections collapses)

F-theory compactification with an n-section (discrete Z_n symmetry)



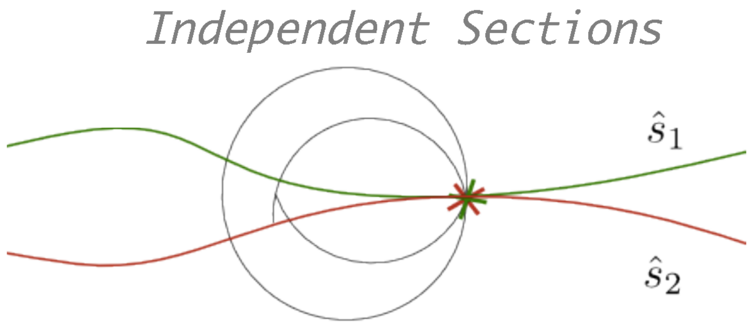
Deformation
(S^3 glues several sections
to a multi-section)

Conifold transition - Geometry

Abelian & Discrete Gauge Symmetry in F-theory

F-theory compactification with n sections (Abelian $U(1)^{(n-1)}$)

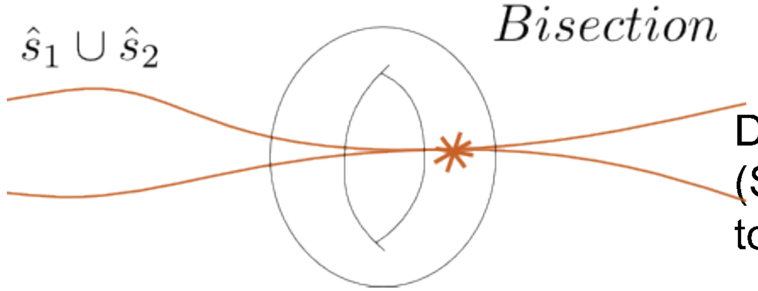
n=2 example



blow-down
(P^1 in the geometry with
multiple sections collapses)

appearance of massless field
ϕ with charge 2

F-theory compactification with an n-section (discrete Z_n symmetry)



Deformation
(S^3 glues several sections
to a multi-section)

massless field acquires VEV
$\langle\phi\rangle \neq 0$
$U(1) \rightarrow Z_2$

Conifold transition - Effective theory

Abelian & Discrete Gauge Symmetry in F-theory

F-theory compactification with n sections (Abelian $U(1)^{(n-1)}$)

↓ blow-down Geometry

connected via conifold transition to

↓ deformation Effective field theory

F-theory compactification with an n-section (discrete Z_n symmetry)

Since Abelian symmetries better understood (c.f., recent works) most efforts focus on the geometry and spectrum of $U(1)^{(n-1)}$, to deduce, primarily via effective field theory, implications for Z_n.

Z_2 [Anderson,Garcia-Etxebarria, Grimm; Braun,Grimm,Keitel; Mayrhofer, Palti, Till, Weigand]

Z_3 [M.C.,Donagi,Klevers,Piragua,Poretschkin] 1502.06953

Tate-Shafarevich Group & Geometry w/ n-section (3-section)

Tate-Shafarevich Group

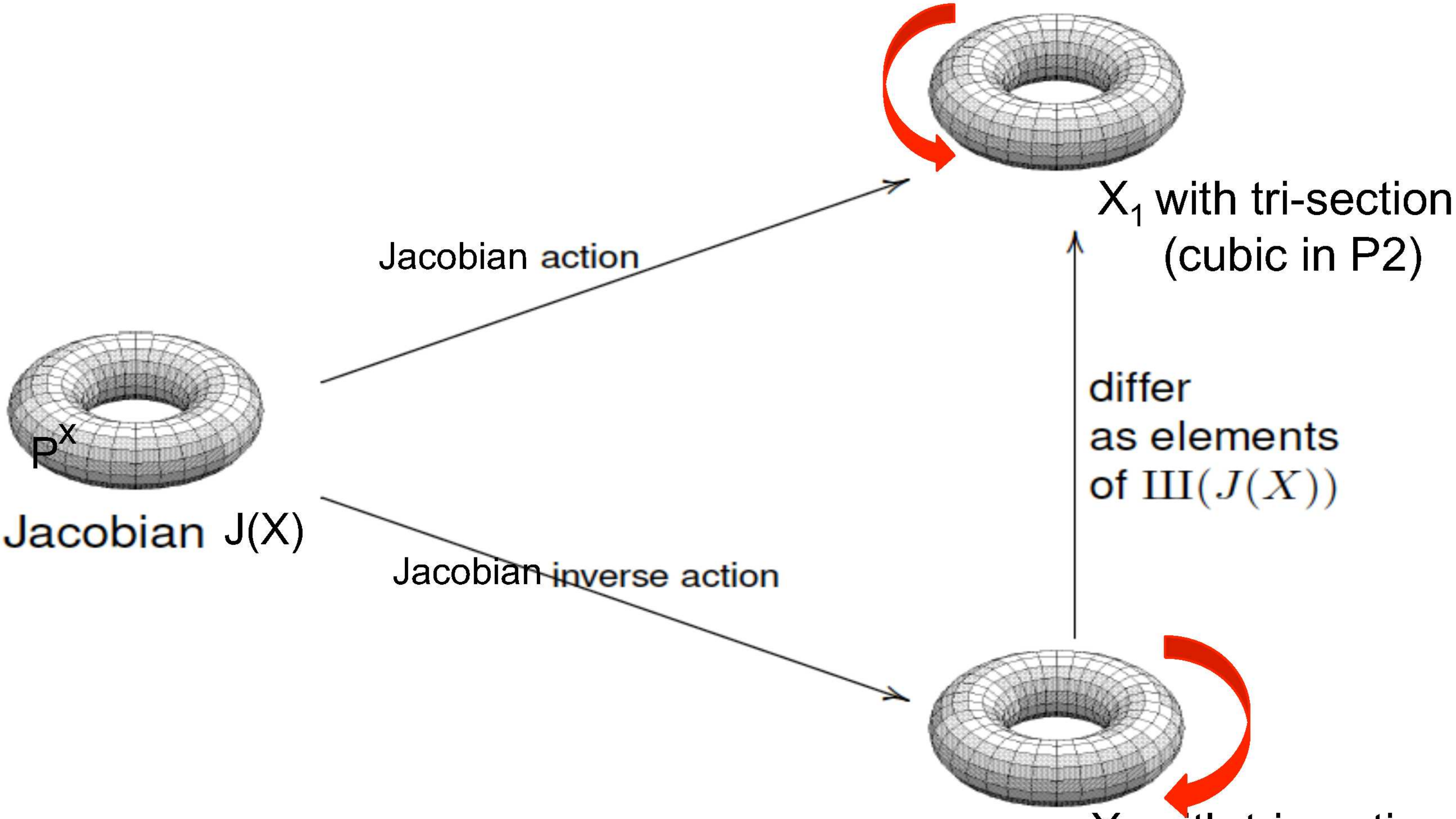
The Tate-Shafarevich group $\text{Ш}(J(X))$
Based on a
collection of Genus-one fibered Calabi-Yau manifolds X_i;
so that the Jacobian fibration of X_i is $J(X)$ }

An element $\mathbf{X_i}$ of $\text{Ш}(J(X))$ consists of a triple: $\mathbf{X_i} = (X_i; f_i; a_i)$:

X_i is a genus-one fibration over a base B
$f_i : X_i \longrightarrow B$ is the fibration map
a_i: $J(X) \times X_i \longrightarrow X_i$ is the Jacobian action

➡ Two geometries with different actions of the Jacobian differ as elements in $\text{Ш}(J(X))$

Tate-Shafarevich group and Z_3



X_1 with tri-section
(cubic in P2)

Jacobian action

differ
as elements
of $\text{Ш}(J(X))$

P^x

Jacobian J(X)

Jacobian inverse action

X_1 with tri-section
(cubic in P2)

Only two geometries $J(X_1)$ and X_1,

but, three different elements of TS group!

[Anderson,Garcia-Etxebarria, Grimm; Braun,Grimm,Keitel; Mayrhofer, Palti, Till, Weigand]

F-Theory/M-Theory Fluxed Circle Compactification → the role of TS (Z_3)

F -theory (Z_3)

6D F-theory $\mathrm{U}(1)_{6d} \xrightarrow{\text{Higgsing of } \Phi_3 \text{ (charge 3)}} \mathbb{Z}_3$

same for X_1 or $J(X_1)$

F/M-theory Fluxed Circle Compactification (Z_3)

$$\begin{tikzcd}
\text{6D F-theory} & \text{U}(1)_{6d} \arrow[r, "\text{Higgsing of } \Phi_3 \text{ (charge 3)}"] \arrow[d, "{S^1 \text{ comp. with } \xi = \int A_1 \text{ [flux-Wilson line]}}"'] & \mathbb{Z}_3 \\
\text{M-theory} & \text{U}(1)_{5d} \times \text{U}(1)_{KK} &
\end{tikzcd}$$

same for X_1 or $J(X_1)$

6D matter
w/ charge 3

$$\Phi_3(x,y) = \sum_{n\in\mathbb{Z}} \phi_n(x) e^{2\pi i n y}, \qquad m(n,\xi) = |3\xi + n|$$

KK expanded tower w/ KK mass

F/M-theory Fluxed Circle Compactification (Z_3)

$$\begin{tikzcd}
\text{6D F-theory}\ U(1)_{6d} \arrow[rr, "\text{Higgsing of } \Phi_3 \text{ (charge 3)}"] \arrow[d, "\begin{array}{c} S^1 \text{ comp.} \\ \text{with} \\ \xi = \int A_1 \\ \text{[flux-Wilson line]} \end{array}"'] & & \mathbb{Z}_3 \arrow[d, "\begin{array}{c} S^1 \text{ comp.} \\ \text{with} \\ \xi = \int A_1^m \\ \text{[massive flux]} \end{array}"] \\
\text{M-theory}\ U(1)_{5D} \times U(1)_{KK} \arrow[rr, "\text{Higgsing of} \begin{cases} \phi_0(\xi = 0) & (3,0) \\ \phi_1(\xi = -\frac{1}{3}) & (-1,2) \\ \phi_2(\xi = -\frac{2}{3}) & (2,-1) \end{cases}"] & & \begin{cases} U(1)\times\mathbb{Z}_3 & [J(X_1)] \\ U(1) & [X_1 \circlearrowright] \\ U(1) & [X_1 \circlearrowleft] \end{cases}
\end{tikzcd}$$

same for X_1 or $J(X_1)$

Charges

6D matter
w/ charge 3

$$\Phi_3(x,y) = \sum_{n\in\mathbb{Z}} \phi_n(x) e^{2\pi i n y}, \qquad m(n,\xi) = |3\xi + n|$$

KK expanded tower w/ KK mass

F-Theory with U(1) & Charge 3 Matter

Can be obtained by Higgsing earlier $U(1)^2$- example or

Directly in geometry: toric blow-up of singular dP_1 [cubic in P^2]

[Klevers, Mayorga-Pena, Oehlmann, Piragua, Reuter]

M-theory: $U(1)_{5D}$ x $U(1)_{KK}$ w/ charge (3,0), (-1,2) & (2,-1) matter

F-Theory with U(1) & Charge 3 Matter

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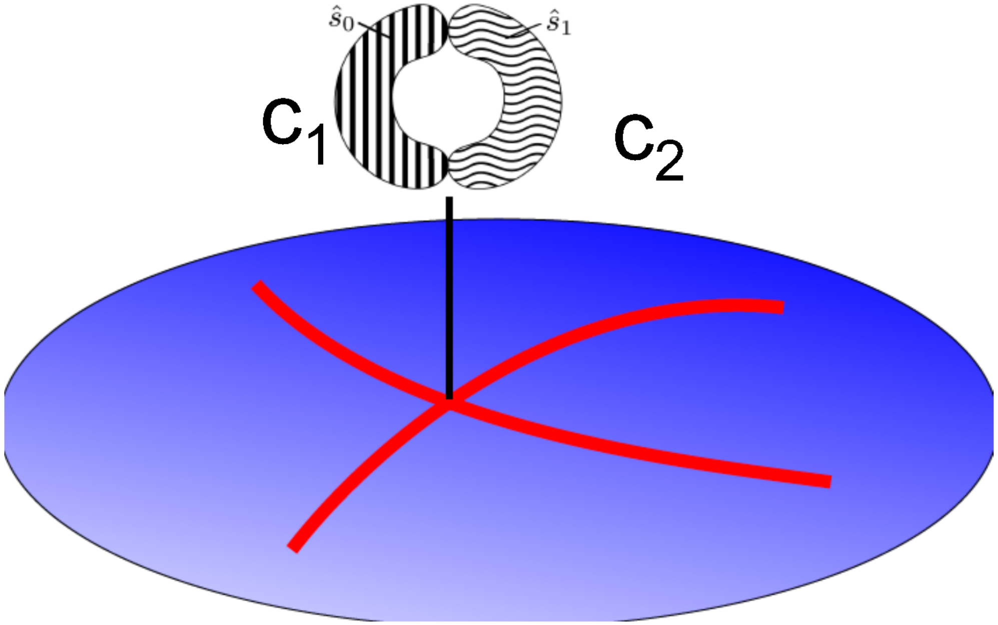
M-theory: $U(1)_{5D}$ x $U(1)_{KK}$ w/ charge (3,0), (-1,2) & (2,-1) matter

		c_1	c_2
$U(1)_{KK}$	$\tilde{S}_0$	-1	2
$U(1)_{5D}$	$\tilde{S}_1$	2	-1
$U(1)_{6D}$	$\tilde{S}_1 - \tilde{S}_0$	3	-3

- ambient space dP_1
- rational zero-section
- additional rational section
- Curves that lead to Higgs field of charge (2,-1) (-1,2) are manifest



In M-theory two correct matter charges for Higgsing (with flux ξ= -1/3; -2/3)

Investigating all M-theory phases

Problem:

- 3 curves needed, but I_2 -fiber only has two components w/ two curves c_1 and $c_{2,}$ with charges (2,-1) and (-1, 2) manifest
- curve with charge (3,0) is not visible in the toric resolution [curve in the class [c_1 + T^2] would have the right charges] Computation of the Gromov-Witten invariant reveals

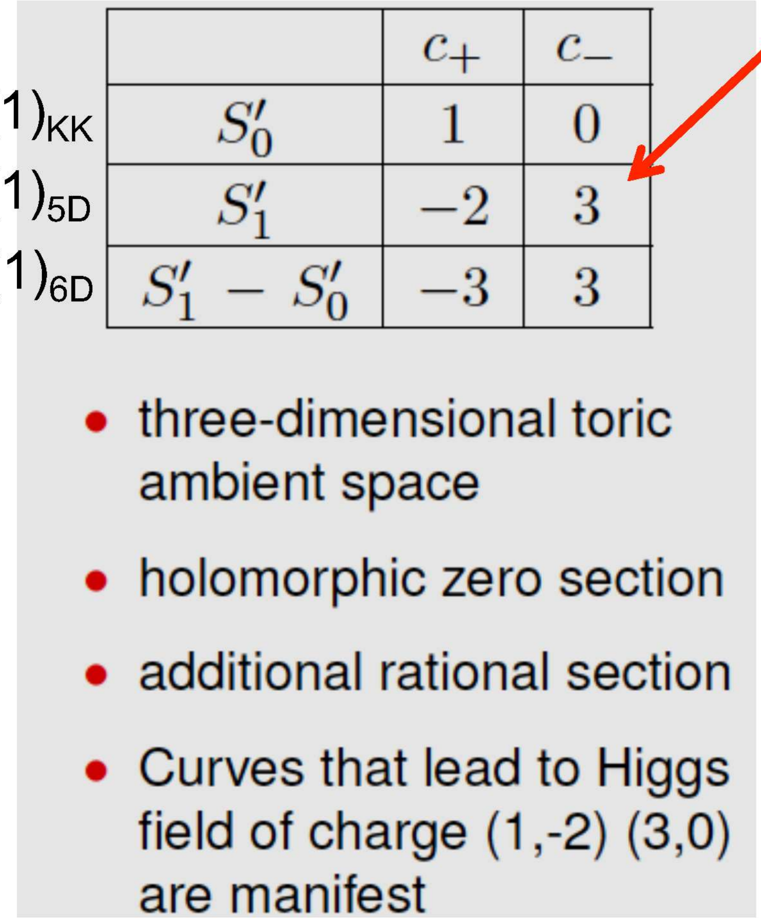
➡ $N_{c1+T^2} = 1$

Where is the geometry with the charge (3,0) curve manifest?

Finding the Third Curve

In a phase where zero section is holomorphic,
related to the original one by a flop transition:

Resolutions of the singular dP_1 via complete Intersection



		c_+	c_-
$U(1)_{KK}$	S_0'	1	0
$U(1)_{5D}$	S_1'	-2	3
$U(1)_{6D}$	$S_1' - S_0'$	-3	3

- three-dimensional toric ambient space
- holomorphic zero section
- additional rational section
- Curves that lead to Higgs field of charge (1,-2) (3,0) are manifest

Correct matter charges
for M-theory Higgsing (with flux ξ=0)
to U(1) x Z_3

Outcome for Z_3

- Explicitly construct the three different curves that yield the fields with desired charges; crucial to consider two different resolutions of the singular dP_1 geometry, related by flop transition.

- One can expect to see at most two matter curves in one resolution explicitly, we can detect the third (hidden) one by computing the Gromov-Witten invariant of the corresponding curve class (which was done).

- Demonstrate that the three different M-theory vacua do not lead to three different geometries but must be distinguished by the action of the Jacobian on the remaining two non-trivial elements of the Tate-Shafarevich group.

Expect to generalize to Z_n n>3.

Summary

- Particle Physics Models: three family Standard, Pati-Salam & Trinification models (tip of the iceberg)

- Abelian Gauge Symmetries: highlight $U(1)^2$ example → generalizations: unHiggsing

- Discrete Gauge Symmetries: highlight Z_3 example via conifold transition. Identify all matter which Higgses to different M-theory vacua ⟺ Tate-Shafarevich group

Work in progress

- First principle calculation of torsional homology for the Jacobian curve of TS geometry…
- Generalization to non-Abelian discrete symmetries: new (``twisted'') torsional homologies…[M.C., Donagi, Poretschkin]